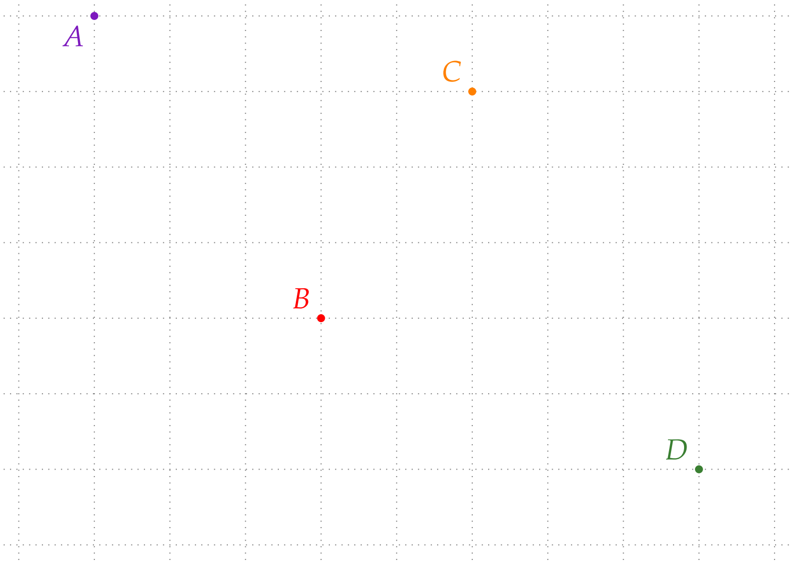
coordinate geometry

distance between two points

Shortest Distance

Find the shortest distance between all pairs of points



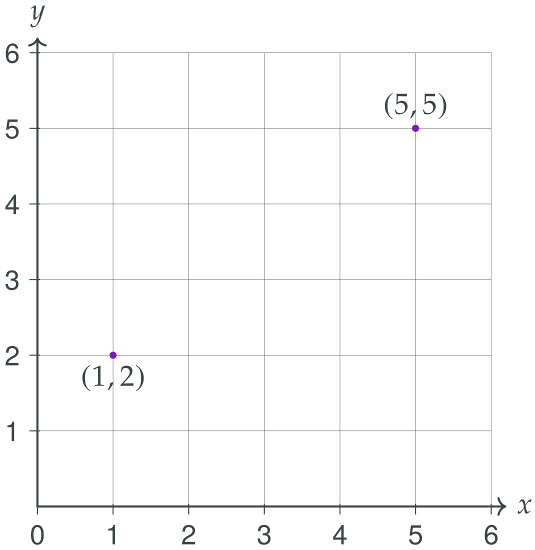
Example

Example 1

Find the shortest distance between the points $(1, 2)$ and $(5, 5)$.

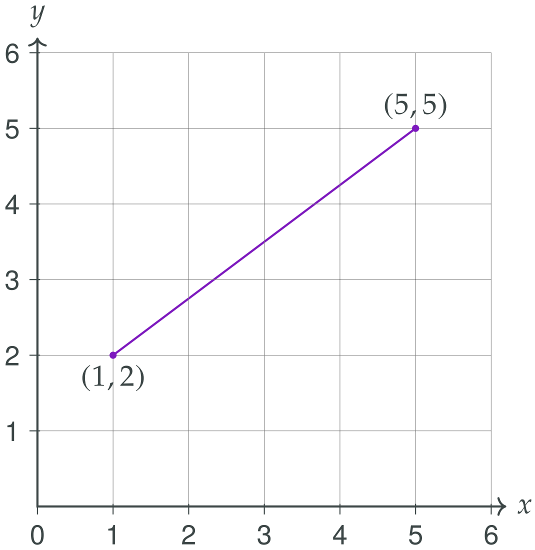
Example

Find the shortest distance numerically



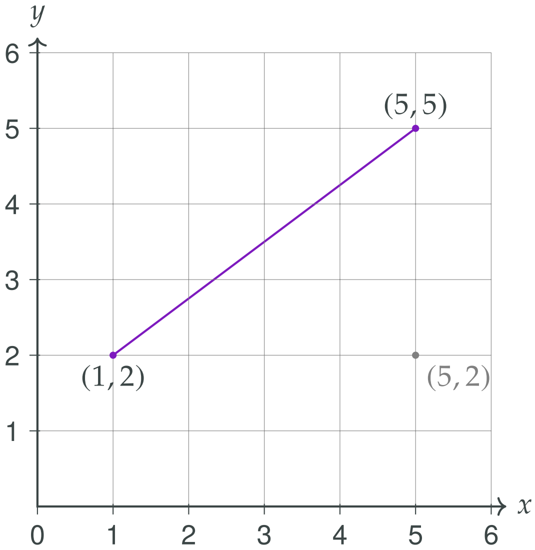
Example

Find the shortest distance numerically



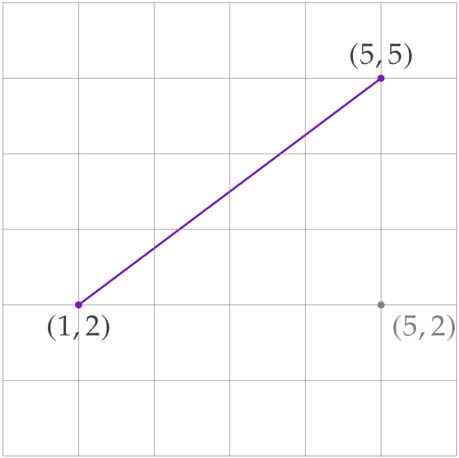
Example

Find the shortest distance numerically



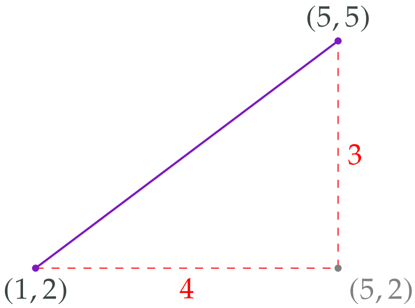
Example

Find the shortest distance numerically

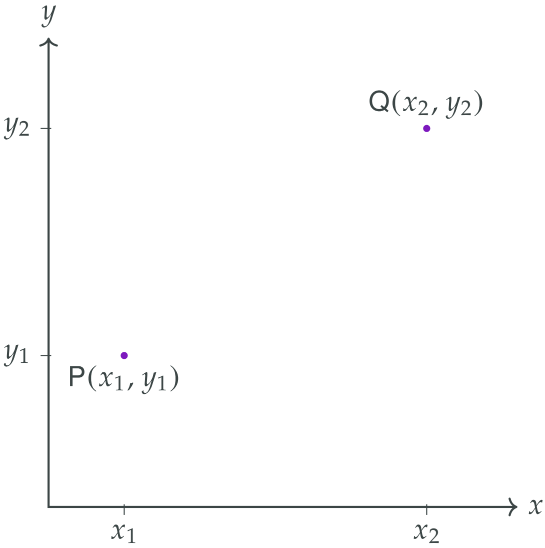


Example

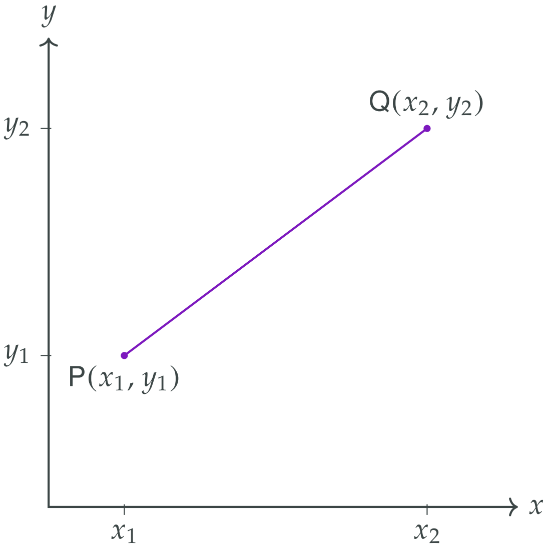
Find the shortest distance numerically



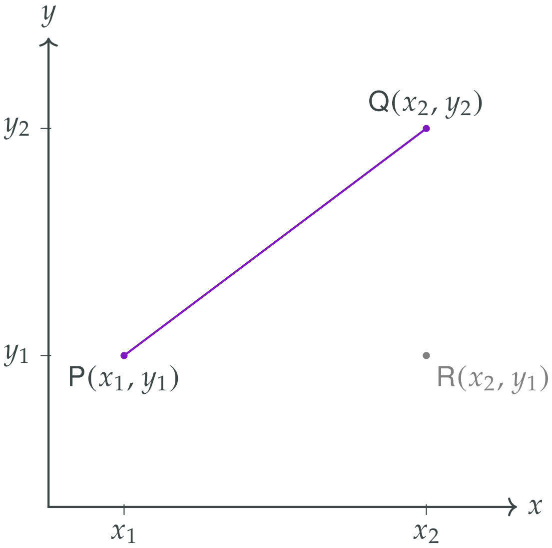
Algebraically



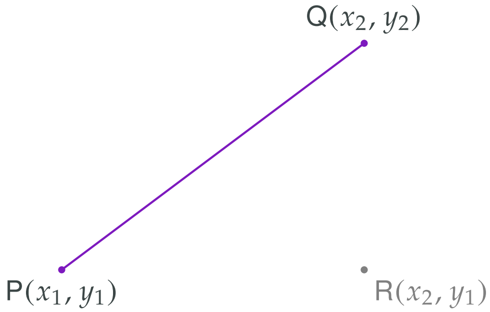
Algebraically



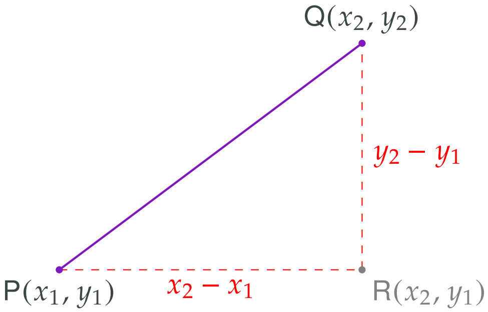
Algebraically



Algebraically



Algebraically



Shortest distance between two points

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

Example 2

Find the shortest distance between $(-2, 3)$ and $(4, -5)$.

Example

Example 2

Find the shortest distance between $(-2, 3)$ and $(4, -5)$.

10

midpoints

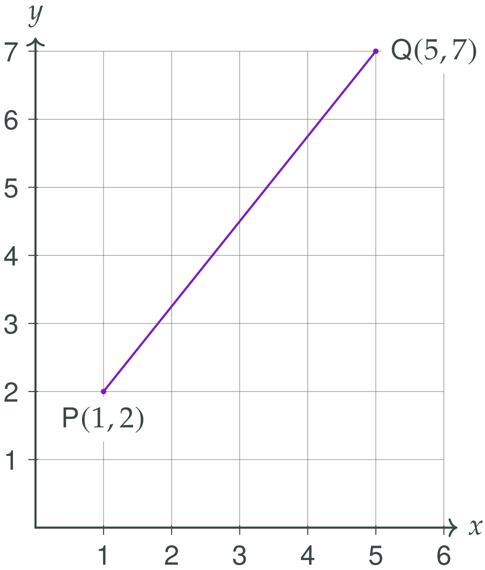
Example

Example 3

Find the midpoint of the line segment connecting the points $W(1, 2)$ and $Z(5, 7)$.

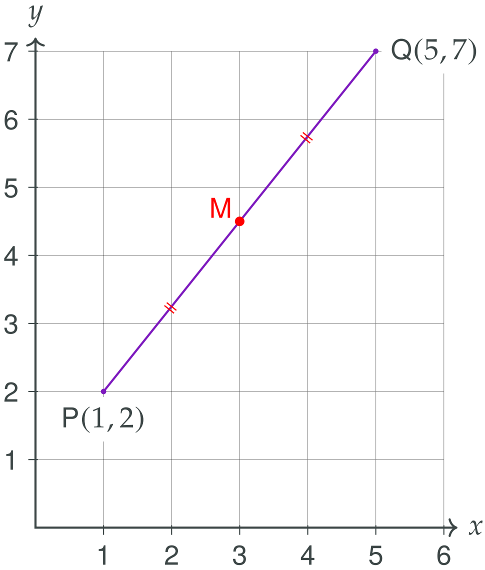
Example

Find the midpoint numerically



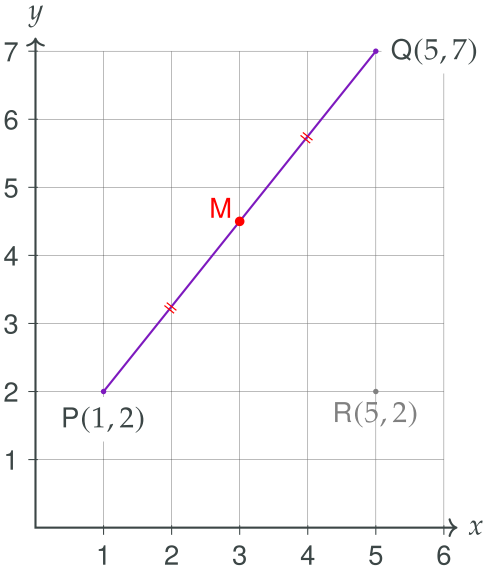
Example

Find the midpoint numerically



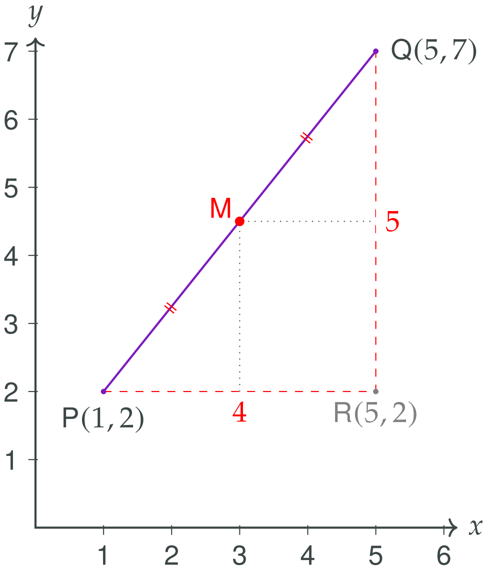
Example

Find the midpoint numerically

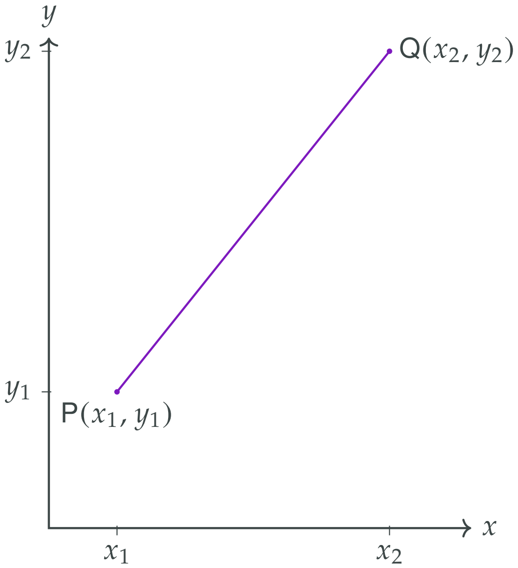


Example

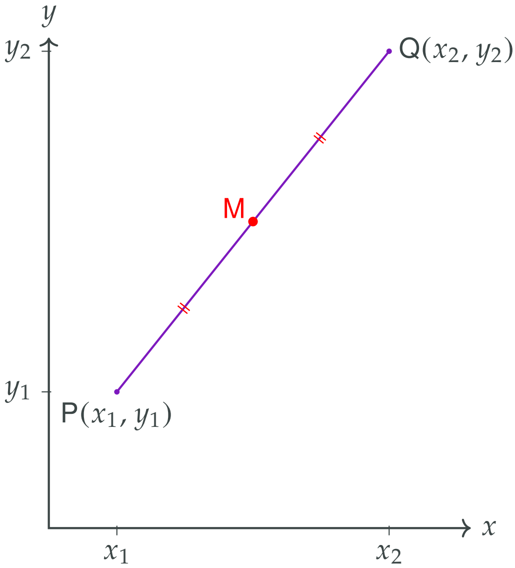
Find the midpoint numerically



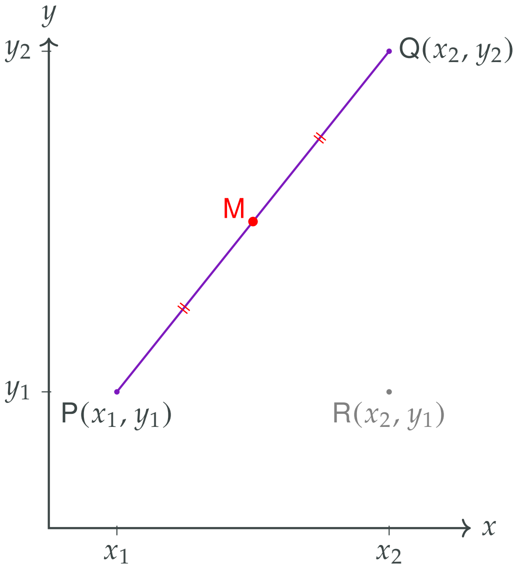
Algebraically



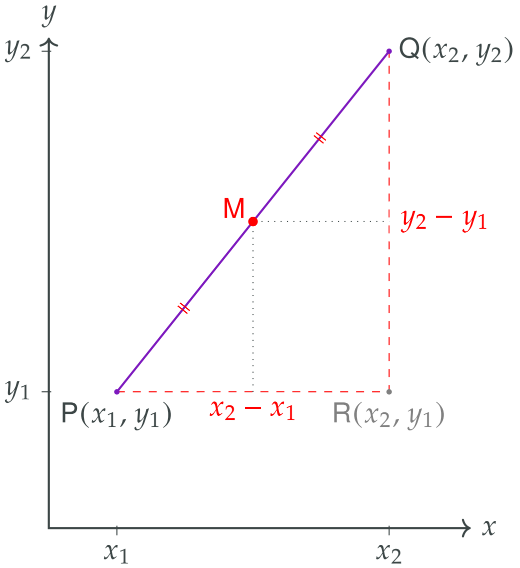
Algebraically



Algebraically



Algebraically



The **midpoint** of a line segment is the average of the two end coordinates

Midpoints

The midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example

Example 4

M is the midpoint of the line segment joining $A(1, -3)$ to $B(3, 4)$.

a. Find the coordinates of M.

b. M is also the midpoint of the line segment CD, where $C(1, 3)$. Find the coordinates of D.

Example

Example 4

M is the midpoint of the line segment joining $A(1,-3)$ to $B(3,4)$.

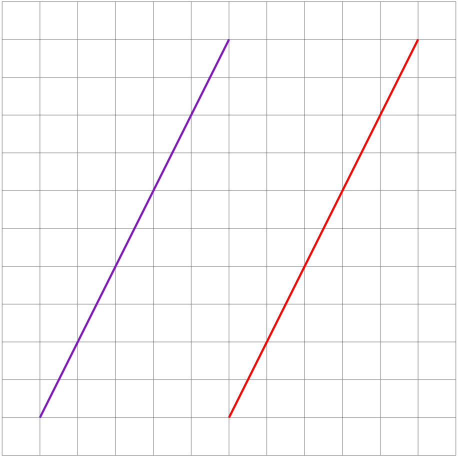
a. Find the coordinates of M.

b. M is also the midpoint of the line segment CD, where $C(1,3)$. Find the coordinates of D.

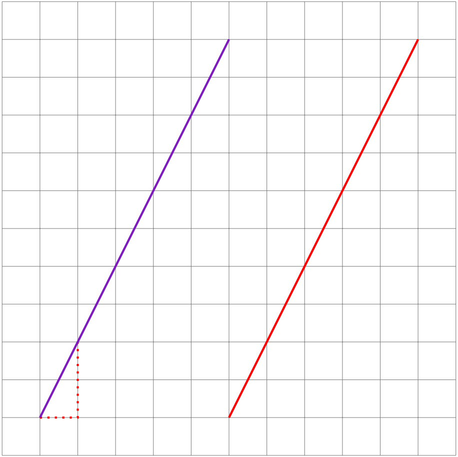
a. $M(2,\frac{1}{2})$ b. $D(3,-2)$

gradients

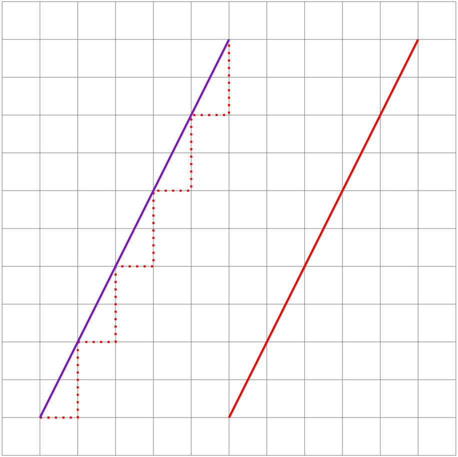
Gradient



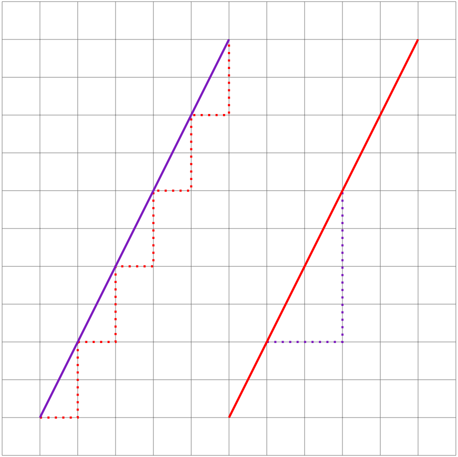
Gradient



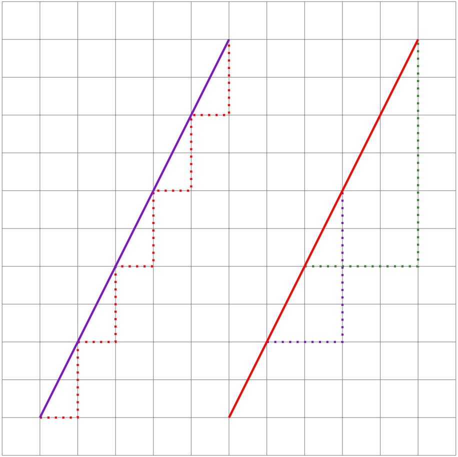
Gradient



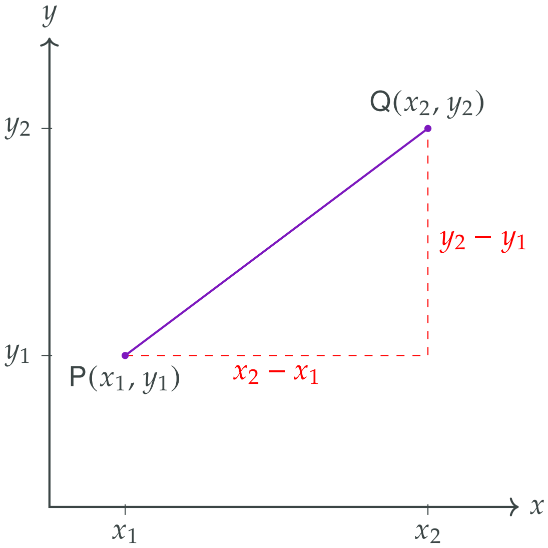
Gradient



Gradient



Gradient



Gradient

$$M = \frac{y_2 - y_1}{x_2 - x_1}$$

Two lines are **parallel** if they have the same gradient.

Example

Example 5

Find the gradient of the lines between

a. $(3, 5)$ and $(5, 9)$

b. $(-2, 2)$ and $(1, -4)$

Example

Example 5

Find the gradient of the lines between

a. $(3, 5)$ and $(5, 9)$

b. $(-2, 2)$ and $(1, -4)$

a. 2 b. -2

Example

Example 6

Show that the points $A(-3,2)$, $B(-2,3)$, $C(-1,-1)$ and $D(-3,-3)$ form a trapezium but not a parallelogram.

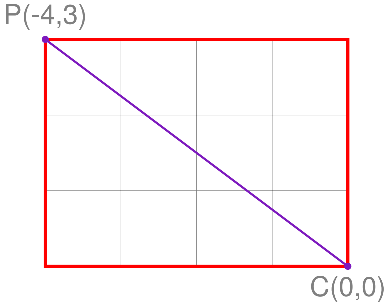
Example

Example 6

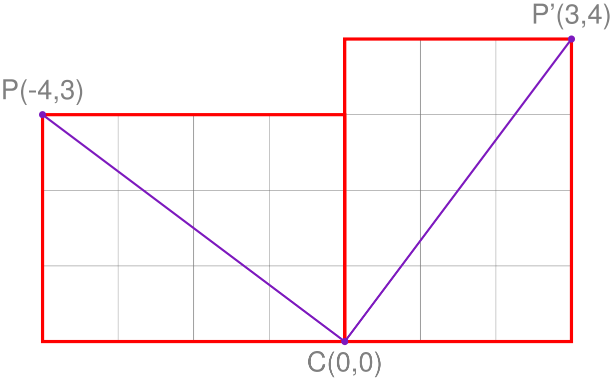
Show that the points $A(-3,2)$, $B(-2,3)$, $C(-1,-1)$ and $D(-3,-3)$ form a trapezium but not a parallelogram.

Gradients: AB = 1 = DC BC = -4 AD = undefined perpendiculars

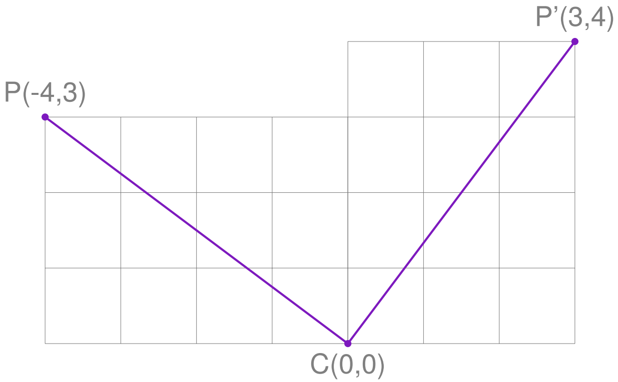
Gradient of perpendicular



Gradient of perpendicular



Gradient of perpendicular



Example

Example 7

Find the gradient of the line that is perpendicular to the line connecting $(1, 5)$ and $(3, 9)$.

Example

Example 7

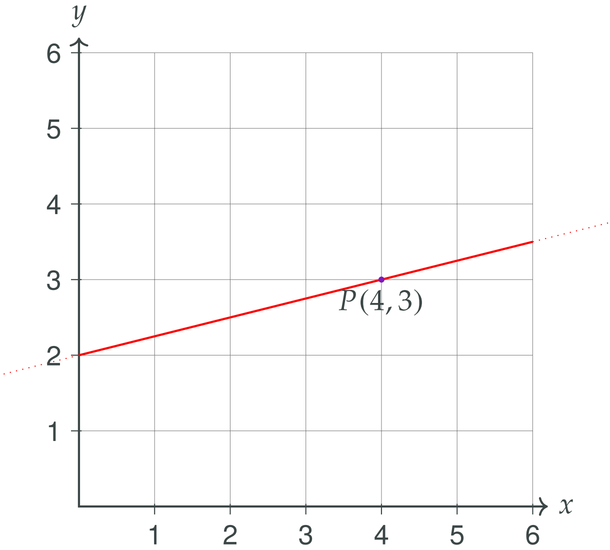
Find the gradient of the line that is perpendicular to the line connecting $(1, 5)$ and $(3, 9)$.

$$m_1 = 2 \implies m_2 = -\frac{1}{2}$$

straight lines

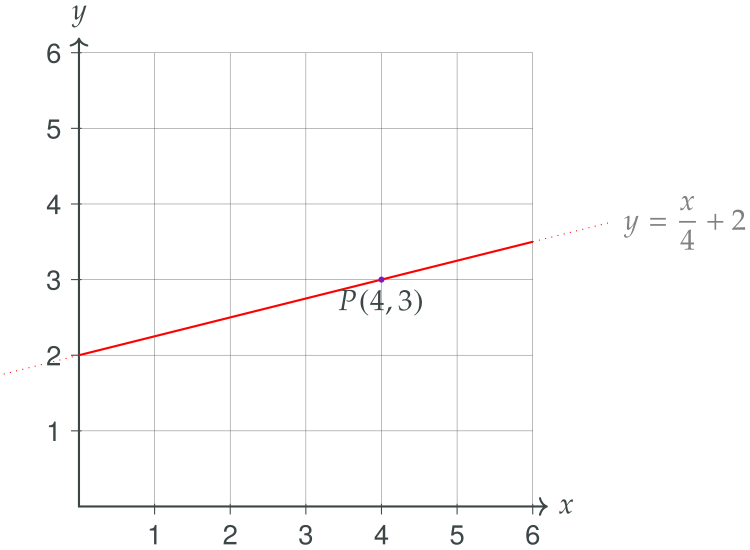
y = mx + c

Gradient and y-intercept (number)



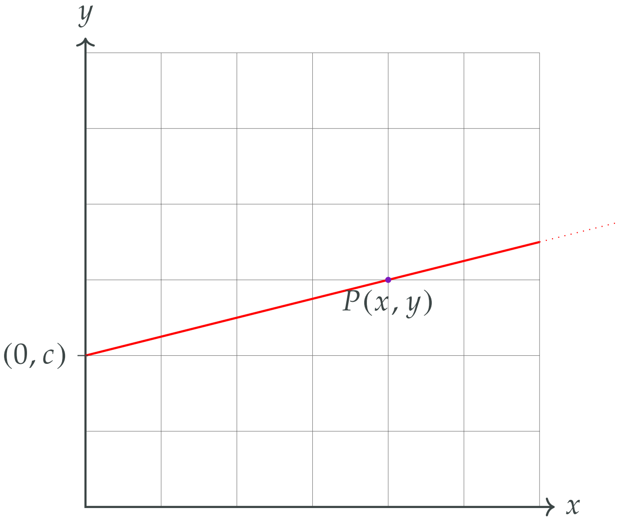
y = mx + c

Gradient and y-intercept (number)



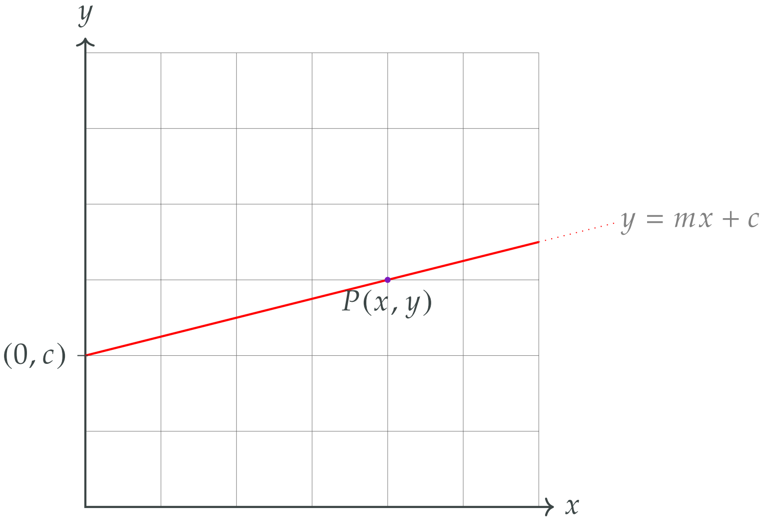
y = mx + c

Gradient and y-intercept (algebra)



y = mx + c

Gradient and y-intercept (algebra)



$y = mx + c$

For equations of the form $y = mx + c$

- m is the **gradient** of the line
- the line crosses the y-axis at $(0, c)$

Example

Example 8

The general equation of a line is $Ax + By + C = 0$. Find the gradient (m) and the y-intercept (c) of the line.

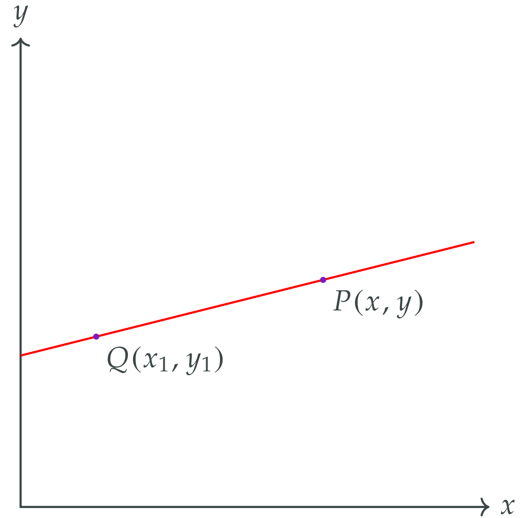
Example 8

The general equation of a line is $Ax + By + C = 0$. Find the gradient (m) and the y-intercept (c) of the line.

$m = -\frac{A}{B} \qquad c = -\frac{C}{B}$

General equation of a line 1

Known gradient and a point on the line



General equation of a line 1

Known gradient and a point on a line

$$y - y_1 = m(x - x_1)$$

Example

Example 9

Find the equation of the line parallel to $y = 3x - 17$ which passes through the point $(2, 5)$.

Example

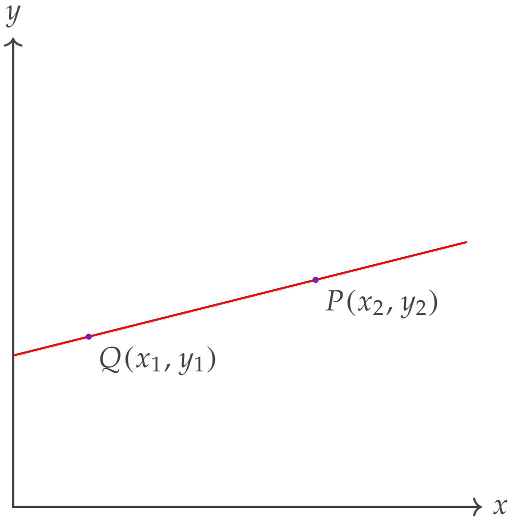
Example 9

Find the equation of the line parallel to $y = 3x - 17$ which passes through the point $(2, 5)$.

$y = 3x - 1$

General equation of a line 2

Two known points on the line



General equation of a line 2

Substituting $\dfrac{y_2 - y_1}{x_2 - x_1}$ into the previous equation of a line and rearranging gives

Two known points on the line

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

Example

Example 10

Find the equation of the line passing through the points $(-1, 1)$ and $(1, 5)$.

Example

Example 10

Find the equation of the line passing through the points $(-1, 1)$ and $(1, 5)$.

$y = 2x + 3$

mixed problems

Example

Example 11

The ends of a line segment are $(v-4w, v+5w)$ and $(v+4w, v-5w)$, where w is positive. Find the

a. length

b. gradient

c. midpoint

of the line segment.

Example

Example 11

The ends of a line segment are $(v - 4w, v + 5w)$ and $(v + 4w, v - 5w)$, where w is positive. Find the

a. length

b. gradient

c. midpoint

of the line segment.

a. $2\sqrt{41}w$ b. $\dfrac{-5}{4}$ c. (v, v)

Extension

The ends of a line segment are $(v-4w, v+5w)$ and $(v+4w, v-5w)$, where w is positive. Pick values for v and w and show the previous result geometrically.

Example

Example 12

The points $P(-2, 1)$, $Q(2, 3)$, $R(0, 0)$ and $S(-4, 2)$. Prove that they form a parallelogram.