

## Reverses

- a) Pick a 2-digit number, e.g. 27
- b) Reverse it (72)
- c) Subtract the smaller number from the larger one ( $72 - 27 = 45$ )
- d) Reverse it (54)
- e) Add the numbers obtained in c) and d) ( $45 + 54 = 99$ )

### 1. Two Digits

$a, b \in \mathbb{N}$  where  $a > b$

e.g.  $a = 7$  and  $b = 2$

$$\begin{array}{r}
 \overset{a-1}{\cancel{a}} \quad \quad \quad \overset{10+b}{b} \\
 - \quad \quad \quad b \quad \quad \quad a \\
 \hline
 (a-1-b) \quad (10+b-a) \\
 + (10+b-a) \quad (a-1-b) \\
 \hline
 9 \quad \quad \quad 9
 \end{array}$$

$$\begin{array}{r}
 \overset{6}{7} \quad \overset{12}{2} \\
 - \quad \quad \quad 2 \quad 7 \\
 \hline
 \quad \quad \quad 4 \quad 5 \\
 + \quad \quad \quad 5 \quad 4 \\
 \hline
 \quad \quad \quad 9 \quad 9
 \end{array}$$

### 2. Three Digits $a, b, c \in \mathbb{N}$

**Case 1:**  $a > c$

e.g.  $a = 3, b = 4, c = 1$

$$\begin{array}{r}
 \overset{a-1}{\cancel{a}} \quad \quad \quad \overset{10+b-1}{\cancel{b}} \quad \quad \quad \overset{10+c}{c} \\
 - \quad \quad \quad c \quad \quad \quad b \quad \quad \quad a \\
 \hline
 (a-1-c) \quad \quad 9 \quad (10+c-a) \\
 + (10+c-a) \quad \quad 9 \quad (a-1-c) \\
 \hline
 1_1 \quad \quad 0 \quad \quad 8 \quad \quad 9
 \end{array}$$

$$\begin{array}{r}
 \quad \quad \quad 3 \quad 4 \quad 1 \\
 - \quad \quad \quad 1 \quad 4 \quad 3 \\
 \hline
 \quad \quad \quad 1 \quad 9 \quad 8 \\
 + \quad \quad \quad 8 \quad 9 \quad 1 \\
 \hline
 1_1 \quad 0_1 \quad 8 \quad 9
 \end{array}$$

**Case 2:**  $a = c$

e.g.  $a = 4, b = 9, c = 4$

$$\begin{array}{r}
 \quad \quad \quad a \quad b \quad c \\
 - \quad \quad \quad c \quad b \quad a \\
 \hline
 \quad \quad \quad 0 \quad 0 \quad 0
 \end{array}$$

$$\begin{array}{r}
 \quad \quad \quad 4 \quad 9 \quad 4 \\
 - \quad \quad \quad 4 \quad 9 \quad 4 \\
 \hline
 \quad \quad \quad 0 \quad 0 \quad 0
 \end{array}$$

### 3. Four Digits $a, b, c, d \in \mathbb{N}$ where $a > d$

**Case 1:**  $b > c$

e.g.  $a = 6, b = 7, c = 3, d = 4$

$$\begin{array}{r}
 \begin{array}{cccc}
 & a & \overset{b-1}{\cancel{b}} & \overset{10+c-1}{\cancel{c}} & \overset{10+}{d} \\
 - & d & c & b & a \\
 \hline
 & (a-d) & (b-1-c) & (10+c-1-b) & (10+d-a) \\
 + & (10+d-a) & (10+c-1-b) & (a-d) & (b-1-c) \\
 \hline
 1_1 & 0 & 8 & 9_1 & 0
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccc}
 & 6 & \overset{6}{\cancel{7}} & \overset{12}{\cancel{3}} & \overset{1}{4} \\
 - & 4 & 3 & 7 & 6 \\
 \hline
 & 2 & 3 & 5 & 8 \\
 + & 8 & 5 & 3 & 2 \\
 \hline
 1_1 & 0 & 8 & 9_1 & 0
 \end{array}
 \end{array}$$

**Case 2:**  $b < c$

e.g.  $a = 6, b = 7, c = 3, d = 4$

$$\begin{array}{r}
 \begin{array}{cccc}
 & \overset{a-1}{\cancel{a}} & \overset{10+}{b} & \overset{c-1}{\cancel{c}} & \overset{10+}{d} \\
 - & d & c & b & a \\
 \hline
 & (a-1-d) & (10+b-c) & (c-b-1) & (10+d-a) \\
 + & (10+d-a) & (c-b-1) & (10+b-c) & (a-1-d) \\
 \hline
 & 9 & 9 & 9 & 9
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccc}
 & \overset{5}{\cancel{6}} & \overset{1}{3} & \overset{6}{\cancel{7}} & \overset{1}{4} \\
 - & 4 & 7 & 3 & 6 \\
 \hline
 & 1 & 6 & 3 & 8 \\
 + & 8 & 3 & 6 & 1 \\
 \hline
 & 9 & 9 & 9 & 9
 \end{array}
 \end{array}$$

**Case 3:**  $b = c$

e.g.  $a = 6, b = 7, c = 7, d = 4$

$$\begin{array}{r}
 \begin{array}{cccc}
 & \overset{a-1}{\cancel{a}} & \overset{10+b-1}{\cancel{b}} & \overset{10+c-1}{\cancel{c}} & \overset{10+}{d} \\
 - & d & c & b & a \\
 \hline
 & (a-1-d) & (10+b-1-c) & (10+c-1-b) & (10+d-a) \\
 + & (10+d-a) & (10+c-1-b) & (10+b-1-c) & (a-1-d) \\
 \hline
 1_1 & 0_1 & 9 & 8 & 9
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccc}
 & \overset{5}{\cancel{6}} & \overset{16}{\cancel{7}} & \overset{16}{\cancel{7}} & \overset{1}{4} \\
 - & 4 & 7 & 7 & 6 \\
 \hline
 & 1 & 9 & 9 & 8 \\
 + & 8 & 9 & 9 & 1 \\
 \hline
 1_1 & 0_1 & 9_1 & 8 & 9
 \end{array}
 \end{array}$$